Building a Methodology for the Comparison and Evaluation of Middle-Years Mathematics Textbooks

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This paper reports on the first stage of the development of a methodology to evaluate mathematics textbooks. The methodology depends on the development of a set of mathematics curriculum principles relevant to the context in which the textbook is used, in this case the state of Queensland and its new years 1 to 10 syllabus. Based on these principles, as well as syllabus content statements and published research, sets of specific curriculum goals are being developed for key topic areas in the curriculum and these goals form the basis for the evaluation of textbooks. The paper reports on the development of the goals and sets of indicators for the topic of ratio and proportion and on a small trial application of the methodology.

In middle-school mathematics, the traditional textbook has long been a key reference for teacher curriculum decision-making and the primary resource for student practice of mathematical techniques. Results from the TIMSS 2002 study indicated that in Australia only 5% of year 8 mathematics teachers did not use a textbook at all, and that about half the year 8 teachers used a textbook as the main lesson resource (Thomson & Fleming, 2004). The relationships among the teacher, the textbook and the implemented curriculum appears to be complex (Remillard, 2000). Tornroos (2004) used the term "potentially implemented curriculum" (p. 2) to describe the role of the textbook and other curriculum materials in the mathematics classroom, this role being an intermediate stage between the intended curriculum and the implemented curriculum. Given the importance of published curriculum materials in mathematics teaching and learning, mainly in the form of textbooks, the establishment of an effective method of evaluation of these materials by their potential users is an important goal.

Many schemes for teacher evaluation of middle-school mathematics textbooks have been proposed over the years. Such schemes often include criteria involving syllabus content coverage, numbers of exercises, layout, use of colour, historical content, instructions for the use of technology, and so on. The state of California even has a criterion of weight for a textbook, the allowable maximum weight increasing with the age of the target students. While these features are of some importance, many evaluations have not focused at all on the essence of the mathematical ideas and the ways the ideas are developed for students. In an earlier paper by the author (Shield, 1998), the underlying "messages" about the nature of mathematics and its teaching and learning conveyed by textbook presentations were explored. In recent years, a number of studies have focused on a more in-depth approach to the evaluation of mathematics textbooks by developing systematic strategies that relate the content of the published materials to the mathematical content and pedagogical practices required by various authorities. The continuing implementation of the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) in the United States has been one stimulus for this kind of approach, given the publication of many so-called "standards-based" textbooks (Martin, Hunt, Lannin & Leonard, 2001).

Project 2061 (Kulm, Morris & Grier, 2000) involved the development of a method to evaluate middle-grades mathematics textbooks with a focus on "their effectiveness in helping students to achieve important mathematical learning goals for which there is broad national consensus" (p. 1). The learning goals and criteria for evaluation were derived from significant documents including the NCTM *Standards* (National Council of Teachers of Mathematics, 1989) and the *Benchmarks for Science Literacy* (American Association for the Advancement of Science, 1994). The evaluation criteria were set out in a number of categories including: Building on student ideas about mathematics; Engaging students in mathematics; Developing mathematical ideas; and Promoting student thinking about mathematics.

Overview of the Project

In Queensland, a new syllabus (Queensland Studies Authority, 2004) is being implemented and mathematics teachers are facing important decisions about the purchase of teaching materials appropriate for the implementation of this syllabus. Australian state education departments provide no more than general guidelines about the selection of appropriate curriculum materials for schools, and there are no specific approval processes. This is not the case in some other parts of the world. For example, the California Department of Education has provided a highly detailed set of criteria for mathematics textbook approval in relation to its recently published Curriculum Framework (California Department of Education, 2005).

This paper describes the first stage of a project to develop a mathematics textbook evaluation strategy that explores the development of the mathematical ideas in the material and assesses the alignment of the pedagogical processes with recommendations from the relevant syllabus documents and research findings. The overall project involves four stages: development and initial validation of an evaluation methodology; trial application of the methodology and further refinement; training of middle-school classroom teachers in the application of the methodology; and reporting the conclusions of teacher evaluations of available mathematics curriculum materials.

Method

The approach to evaluating mathematics textbooks in the present study has been influenced by the methods of Project 2061 (Kulm et al., 2000). The essence of the Project 2061 approach is to examine textbooks in relation to a set of principles derived specifically for the context in which the materials are to be used. For the present study, a set of mathematics curriculum principles was developed from the explanatory material in the new *Years 1to10 Mathematics Syllabus* (Queensland Studies Authority, 2004). To assist in the explication of the principles, the syllabus statements were compared with statements in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000). There is very close agreement between the two documents in most areas. The mathematics curriculum principles are then being used, in conjunction with syllabus content statements and published research related to that content, to develop sets of content-specific curriculum goals at the middle-years level.

It is generally impractical and unnecessary to examine in detail the development of every mathematical topic presented in a series of textbooks, or even one book. Several key topic areas are being examined and specific curriculum goals developed for each of them. In this paper, the topic of ratio and proportion from the "Number" strand of the syllabus is used to illustrate the method of developing the specific curriculum goals. Content-specific curriculum goals are being derived for other topics including functions (Patterns & algebra strand) and relationships between lines (Space strand).

A set of "indicators" has been written for each content-specific curriculum goal. The indicators describe specific cues that might be present in a text that demonstrates affinity with the requirements of the QSA syllabus and with a deeper understanding of mathematics. Feedback on the draft curriculum goals was obtained from two mathematics curriculum specialists. The specialists were presented with a set of content-specific curriculum goals and indicators, supported by an elaborated set of general curriculum principles specific to the Queensland syllabus and a brief outline of research into the teaching and learning of proportion. Their feedback led to minor modifications to the wording of both the general curriculum goals and the specific goals for proportion but there was overall agreement that the framework was a useful representation of the position taken by the syllabus. The paper also reports briefly on a preliminary application of the framework to one year 8 Mathematics textbook.

Developing the Curriculum Goals for Proportion

Mathematics Curriculum Principles

The mathematics curriculum principles are specific to the Queensland context, being derived from an examination of the *Years 1 to 10 Mathematics Syllabus* (Queensland Studies Authority, 2004). The brief elaborations of the principles provided here refer also to the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000).

- The curriculum should be connected and coherent. Although normally described in strands such as Number and Algebra, key mathematical ideas should be integrated and interconnected across the strands. The Syllabus (Queensland Studies Authority, 2004) recognises that "the mathematical knowledge developed is richly interconnected" (p. 10) and has mathematics "arranged in five strands for organisational convenience" (p. 14), noting that "each strand includes interconnecting topics" (p. 14). According to the Standards (National Council of Teachers of Mathematics, 2000): "The interconnections should be displayed prominently in the curriculum and in instructional materials and lessons" (p. 15).
- Students can and should learn with understanding. The Syllabus aims to develop a "knowledgeable person with deep understanding" (p. 5) and advocates the development of understanding "through active engagement in mathematical investigations and in communicating their thinking and reasoning" (p. 5). Children learn mathematics with understanding by "actively building new knowledge from experience and prior knowledge" (Standards, p. 20).
- Students should be engaged in working mathematically through investigations and problem solving. The Syllabus discusses "making sense of life experiences or seeking solutions to problems" (p. 4) and states that: "Positive dispositions towards mathematics and active engagement with mathematical tasks are integral to thinking, reasoning and working mathematically" (p. 4). According to the Standards: "Problem solving is an integral part of all mathematics learning" (p. 52).
- Students should communicate with and about mathematics. The Syllabus aims to assist each student to become an "effective communicator" (p. 6). The Standards

regards communication as “an essential part of mathematics and mathematics education” (p. 60).

- Students should experience multiple representations of mathematical ideas. The Syllabus (QSA, 2004) requires students to “read, view, analyse and interpret the mathematics represented by text, pictures, symbols, tables, graphs and technological displays” (p. 7) and includes many specific references to the use of representations in its learning outcomes, for example, “students interpret and compare different representations of linear and simple non-linear functions and solve the related problems” (p. 26). “The ways in which mathematical ideas are represented is fundamental to how people can understand and use those ideas” (Standards, p. 67).
- Students’ mathematical learning should involve the integral use of relevant electronic technologies. The Syllabus (QSA, 2004) does not specifically highlight the use of technology as a goal in its own right. Rather, it reflects a view of technology as integral to everything that students do in statements such as: “select and use relevant mathematical knowledge, procedures, strategies and technologies to analyse and interpret information” (p. 4) and: “Learning is enhanced by the use of a range of technologies” (p. 11). The Standards includes: “The Technology Principle” (p. 24) and emphasises the role of technology in enhancing students’ learning and its effect on the nature of the curriculum itself.

Ratio and Proportion in the Middle-school Curriculum

There is insufficient space to provide a detailed review of the extensive literature on the learning and teaching of proportion and related concepts. The key findings reported below arise mainly from the work of Vergnaud (1983), Lamon (1994; 1999), Post, Behr and Lesh (1988), and Cramer, Post and Currier (1992).

The following example illustrates the notion of “measure spaces” (Vergnaud, 1983) and “within” and “between” solution strategies (Lamon, 1999). This is an example of a typical “missing value” problem.

> A cordial drink is made up of a mixture of 1 part cordial to 5 parts water. How much water needs to be mixed with 40 mL of cordial to make a drink of the required strength?

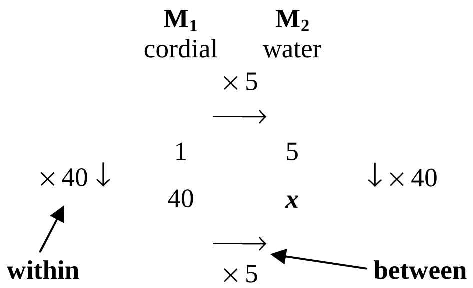

Figure 1. Solution strategies for a proportion problem.

In summary, the extensive research into the teaching and learning of proportion leads to the following recommendations.

- Proportion is not a separate topic. Proportional thinking is the key concept in a wide range of mathematical contexts and problem types including ratio, rate, gradient and scale.

- Proportional thinking is multiplicative and requires an understanding of inverse operations.
- Proportional situations involve two quantities or “measure spaces” that may have the same or different units.
- Solving proportion problems requires the flexibility to work “within” and “between” the measure spaces.
- Proportional situations can be represented in multiple ways, including tables, number lines, linear graphs and equations.
- Solving “missing-value” problems using algebraic manipulation of a proportion equation should be delayed until a sound understanding of proportion has been developed.

Specific Curriculum Goals for Proportion

The *Syllabus* (Queensland Studies Authority, 2004) has the content stated in terms of core learning outcomes and core content, set in a frame of 6 levels. Levels 4 to 6 are generally applicable to middle-school aged learners. The following is a summary of the outcomes and core content from levels 4 to 6 related to proportion.

> <u>N 4.3</u> Students identify and solve multiplication and division problems involving whole numbers, decimal fractions, percentages and rates, selecting from a range of computation methods, strategies and known number facts.
>
> <u>N 5.3</u> Students identify and solve multiplication and division problems involving positive rational numbers, rates, ratios and direct proportions using a range of computation methods and strategies (inverse, calculations involving everyday rates, simple everyday ratios, symbol for ratio, calculations with direct proportion including graphical representations).
>
> <u>N 6.3</u> Students identify and solve multiplication and division problems involving positive rational numbers, rates, ratios and direct and inverse proportions using a range of computation methods and strategies (comparisons of rates expressed in various forms, direct proportion, inverse proportion)

The following specific curriculum goals and indicators were derived by considering the outcomes stated in the syllabus in relation to the research findings on the learning of proportion as well as the curriculum principles derived from the syllabus and other sources.

Learning material that promotes an effective teaching approach to the topic of proportion in Queensland middle-schools should have the following characteristics.

- Highlights common multiplicative structures and proportional thinking (MS)
- Indicators: (a) multiplicative nature of the “comparison” made clear and distinguished from additive comparison; (b) common multiplicative structure made explicit in different problem types such as ratio, rate, scale, chance, gradient; (c) use of proportional thinking to solve problems made explicit in worked examples.
- Makes explicit the connections of the mathematical ideas being developed with related mathematical ideas (CON)
- Indicators: (a) common methods used for similarly structured problems and the similarities between solution methods made explicit; (b) clear links made with ideas of fraction and equivalence, with identification of part-whole and part-part relationships; (c) applications are genuine use of the appropriate concepts in the students’ environment.

- Engages students in appropriate investigative and problem solving experiences (I&PS)
- Indicators: (a) investigation and problem solving used in the development of concepts and not just as applications; (b) links with existing ideas and the building of new concepts made explicit; (c) situations used are appropriate to the target students.
- Engages students in reflecting on and communicating about their mathematical understanding (R&C)
- Indicators: (a) communicating in oral and written forms is part of investigation and problem solving experiences; (b) communication tasks likely to promote effective student reflection; (c) communication tasks include possibilities for effective feedback from teacher or other students.
- Makes effective use of a range of representations (REP)
- Indicators: (a) uses tables, linear graphs, number lines to represent proportional situations; (b) uses representations to build understanding of the multiplicative structure of the problem types; (c) delays the introduction of the formal "proportion equation" until extensive experience has been gained with other representations.
- Incorporates the use of technologies (TEC)
- Indicators: (a) methods for using simple calculator to solve proportion problems made explicit; (b) spreadsheet used to generate data for proportional variables; (c) graphing technologies used to investigate data.

Preliminary Application of the Framework

In order to further develop the "indicators" for the specific curriculum goals for proportion, the author has conducted trial applications to newly published year 8 textbooks, and some brief conclusions from those continuing trials are provided here. One textbook by Brodie and Swift (2004) is discussed. Each of the six specific curriculum goals was explored in one chapter entitled "Fractions and ratios" and another called "Money and percentage". Because of space limitations, only the first two curriculum goals are discussed.

Multiplicative Structures (MS)

The following statements come from a section "Ratios and rates".

> A **rate** is obtained by dividing one quantity by a different, related quantity. It is sometimes useful to express rate as a fraction. (p. 177)

> Rates can be used to calculate the first quantity from the second. In this case we use multiplication to reverse division. (p. 177)

Although not explained beyond these statements, the text makes the multiplicative nature of rate explicit. Ratio is defined on the next page of the text with no mention of a multiplicative comparison.

> A **ratio** compares quantities of the *same kind* in a definite order. (p. 178)

The worked examples that accompany this section on rate and ratio make no use of proportional thinking, concentrating on using the formula for speed and on cancelling down by finding common factors. The following statement that clearly links ratio and rate to proportional thinking is made half-way through the 8 worked examples on ratio and rate, but the idea is not used anywhere in the worked examples.

> Rates and ratios are closely related. For example, a speed of 5 m/s means that the object travels 20 m in 4 s or 30 m in 6 s. The ratio of the distances, 20 : 30, is the same as the ratio of the times, 4 : 6, because they both simplify to give 2 : 3. (p. 179)

The nature of proportion is discussed in a section entitled "Direct proportion", after students practice several pages of problems involving rates and ratios. In this section, speed is again featured with both a table and linear graph showing the proportionality. However, the worked example goes back to the distance/time formula for finding the answers. The ratio example uses a proportion equation but there is no explanation of why one should: "Multiply by the 2 to get the value" (p. 185).

Connections (CON)

The methods used to solve similarly structured problems involving fractions, ratios and percents, are very similar in that all are stated in terms of similar algorithms. However, the similarity of the methods, particularly the idea of equivalent fractions, is not mentioned. The connection of ratio with fraction is made in stating the three ways a ratio can be written, including "in **fraction** notation with the second quantity as the denominator" (p. 178). There is no mention of the difference between part-part and part-whole relationships, the only mention of parts and wholes being in a standard "sharing" problem. A clear connection is made between the concepts of fraction and percent, again without any explicit use of the idea of equivalent fractions.

> In other words, percentages are really fractions with denominators of 100. (p. 237)

The use of the Indicators

In their use so far, the indicators have been useful in highlighting features in the text to look for. Similar types of comments were made in regard to the other proportion goals. One issue that has arisen from the trial applications, that was not captured in the initial indicators, concerns the idea of "coherence" incorporated in the first curriculum principle. In the text discussed above, many of the ideas of multiplicative structure and possible connections could be found. However, there was a sense of a lack of genuine integration and cohesiveness in the development of the ideas, with the underlying idea of proportion not clearly explicated. Further thought is being given to ways of expressing the idea of coherence in the indicators. Another issue to be explored later in the project concerns the training of teachers to apply the indicators. Having developed the curriculum principles and indicators, the author has a thorough knowledge of them before attempting their application, which will not be the case with teachers.

Conclusion

While the *Syllabus* (2004) advocates a connected and integrated approach to mathematics, with the concept of proportion and related concepts of rate and ratio cross-referenced in different strands, little guidance is provided to teachers as how that might be achieved. Textbook authors and publishers, if they truly understand the requirements, are in a position to assist teachers to achieve the goals of the syllabus. The project, the beginnings of which are reported in this paper, aims to provide teachers with a highly informed and practical method of selecting print resources appropriate to their needs.

Learning to use the method should provide teachers with significant professional development as they increase their familiarity with the new syllabus and with the teaching and learning of important mathematical ideas.

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